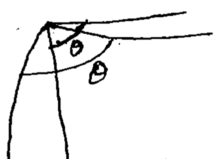
SLOPE DEFLECTION METHOD

- Using this method basic unknowns like slopes and deflections of Joints can be Calculated.
- Moments at the ends of a member is first written in terms of unknown slopes and deflections of the end Joints.
- Considering Joint equilibrium conditions, a set of equations are formed and solution of these equations gives unknown slopes and deflections.
- Then End moments of individual members are determined.
- Since it involves solution of simultaneous equations, a problem with more than three unknowns is considered as difficult for hand calculation.
- Development of this method in the matrix form has led to the " Stiffness matrix method".

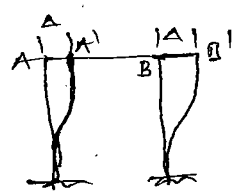


Assumptions :- 1. All Joints are Rigid,

2. Distortions due to axial deformations are neglected

$AA' = BB' = \Delta$

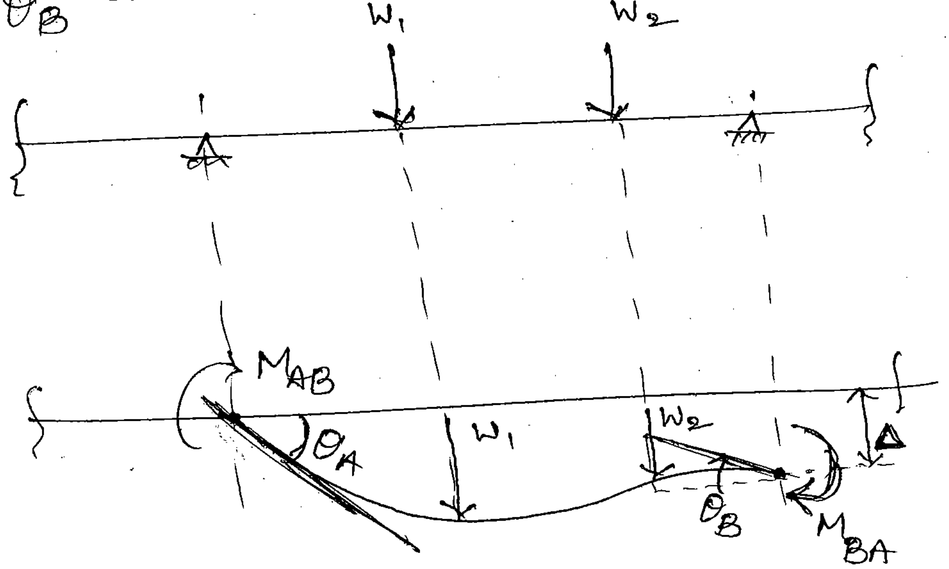
3. Shear deformations are neglected



Sign conventions:-

1. C.W moments are +ve ; ACW moments are -ve
2. C.W Rotations " +ve ; ACW Rotations " -ve
3. Settlement 'Δ' is +ve if Right hand Side Support settles down
" " -ve " left " " .

Derivation: Let AB, shown below, be a member of a Rigid structure. After loading it undergoes deformations. Final moments at end A and end B are M_{AB} and M_{BA}. Now our aim is to derive the relationship between these final end moments and their displacements θ_A, θ_B and Δ.



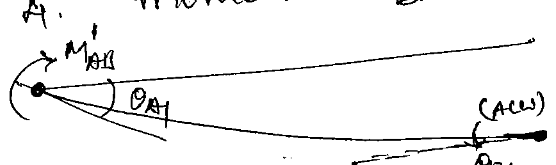
The solution & development of final end moments can be visualised in the following stages.

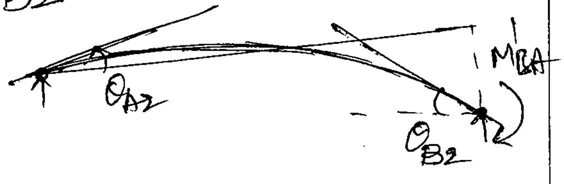
1. Due to given loadings end moments M_{FAB}, M_{FBA} develop without any rotations at ends. Hence called fixed end moments.

2. Settlement Δ takes place without any rotations at ends. This is similar to the settlement of supports in a fixed beam. Hence moments developed are $\frac{6EI\Delta}{L^2}$

3. moment M'_{AB} comes into play in SSB to cause end rotations θ_{A1}, θ_{B1} at A & B resp.

4. moment M'_{BA} → θ_{A2}, θ_{B2}.





$$\therefore \theta_{A1} = \frac{M'_{AB} L}{3EI} \qquad \theta_{B1} = \frac{M'_{AB} L}{6EI}$$

$$\theta_{A2} = \frac{M'_{BA} L}{6EI} \qquad \theta_{B2} = \frac{M'_{BA} L}{3EI}$$

$$\therefore \theta_A = \theta_{A1} - \theta_{A2} = \frac{M'_{AB} L}{3EI} - \frac{M'_{BA} L}{6EI}$$

$$\theta_B = \theta_{B2} - \theta_{B1} = \frac{M'_{BA} L}{3EI} - \frac{M'_{AB} L}{6EI}$$

$$2\theta_A + \theta_B = \frac{2M'_{AB} L}{3EI} - \cancel{\frac{M'_{BA} L}{3EI}} + \cancel{\frac{M'_{BA} L}{3EI}} - \frac{M'_{AB} L}{6EI}$$

$$= \frac{M'_{AB} L}{2EI} \Rightarrow M'_{AB} = \frac{2EI}{L}(2\theta_A + \theta_B)$$

$$\text{Lly} \quad M'_{BA} = \frac{2EI}{L}(2\theta_B + \theta_A)$$

due to Support Settlement

$$\therefore M_{AB} = M_{FAB} - \frac{6EI\Delta}{L^2} + M'_{AB} = M_{FAB} + M'_{AB} + M''_{AB}$$

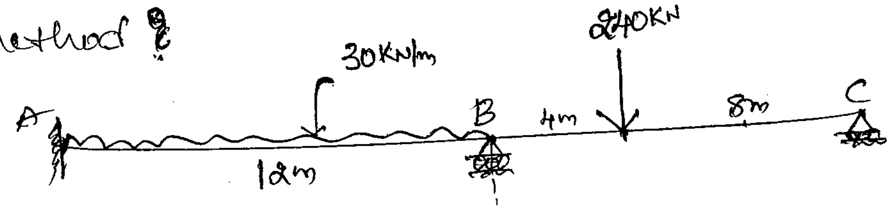
$$= M_{FAB} + \frac{2EI}{L}(2\theta_A + \theta_B) - \frac{6EI\Delta}{L^2}$$

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left[(2\theta_A + \theta_B) - \frac{3\Delta}{L}\right] \quad \text{—①}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left[(2\theta_B + \theta_A) - \frac{3\Delta}{L}\right] \quad \text{—②}$$

① & ② Are Slope deflection Equations

A continuous beam ... over rollers at B and C.. AB = BC = 12m. Beam carries a UDL of 30 KN/m over AB and a point load of 240 KN at a distance 4m from B on span BC. B has a settlement of 30mm. $E = 2\times10^5\ N/mm^2$, $I = 2\times10^9\ mm^4$. Analyse the beam by slope deflection method ?



Sol:

writing FEMs:
$$M_{FAB} = -\frac{30\times12^2}{12} = -360\ KNm$$

$$M_{FBA} = 360\ KNm.$$

$$M_{FBC} = -\frac{240\times4\times8^2}{12^2} = -426.67\ KNm.$$

$$M_{FCB} = +\frac{240\times8\times4^2}{12^2} = 213.33\ KNm$$

$$\therefore\ M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A+\theta_B-\frac{3\Delta}{L}\right)$$

$$= -360 + \frac{2\times4\times10^5}{12}\left[2\theta_A+\theta_B-\frac{3\times\left(\frac{30}{1000}\right)}{12}\right]$$

$$= -360 + \frac{2\times10^5}{3}\left[2\theta_A+\theta_B-\frac{3}{400}\right)$$

$$M_{BA} = 360 + \frac{2\times4\times10^5}{12}\left(2\theta_B+\theta_A-\frac{3\times\left(\frac{30}{1000}\right)}{12}\right]$$

$$= 360 + \frac{2\times10^5}{3}\left[\theta_A+2\theta_B-\frac{3}{400}\right)$$

$$M_{BC} = -426.67 + \frac{2\times4\times10^5}{12}\left[2\theta_B+\theta_C+\frac{3\times\left(\frac{30}{1000}\right)}{12}\right]$$

$$= -426.67 + \frac{2\times10^5}{3}\left[2\theta_B+\theta_C+\frac{3}{400}\right]$$

$$M_{CB} = 213.33 + \frac{2}{3}\times10^5\left[\theta_B+2\theta_C+\frac{3}{400}\right]$$

Substituting, $\theta_A = 0$ and $\Sigma M_B = 0$,

$M_{BA} + M_{BC} = 0 \longrightarrow$

$$360 + \frac{2\times10^5}{3}\left[\cancel{\theta_A}^{0} + 2\theta_B - \frac{3}{400}\right] + (-426.67) + \frac{2\times10^5}{3}\left[2\theta_B + \theta_C + \frac{3}{400}\right] = 0$$

$$4\theta_B + \theta_C = \frac{66.67\times3}{2\times10^5} \longrightarrow (1)$$

$$M_C = 0 \Rightarrow M_{CB} = 0 \Rightarrow 213.33 + \frac{2\times10^5}{3}\left(\theta_B + 2\theta_C + \frac{3}{400}\right) = 0$$

$$\frac{2\times10^5}{3}(\theta_B + 2\theta_C) = -713.33$$

$$-\theta_B + \cancel{2\theta_C} = +\frac{713.33\times3}{2\times10^5}$$

$$2(4\theta_B) + \cancel{2\theta_C} = \frac{66.67\times6}{2\times10^5}$$

$$7\theta_B = \frac{2540}{2\times10^5} \Rightarrow \boxed{1.814\times10^{-3} = \theta_B}$$

$$\theta_C = \frac{66.667\times3}{2\times10^5} - 4\times1.814\times10^{-3}$$

$$\boxed{\theta_C = -6.256\times10^{-3}}$$

$$\therefore\ M_{AB} = -360 + \frac{2\times10^5}{3}\left[\left(0 + 1.814\times10^{-3}\right) - \frac{3}{400}\right] = \underline{-739.067\ \text{KNm}}$$

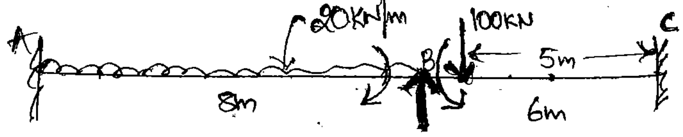
$$M_{BA} = 360 + \frac{2\times10^5}{3}\left[0 + 2\times1.814\times10^{-3} - \frac{3}{400}\right] = \underline{101.867\ \text{KNm}}$$

$$M_{BC} = -426.67 + \frac{2\times10^5}{3}\left[2\times1.814\times10^{-3} + (-6.256\times10^{-3}) + 7.5\times10^{-3}\right]$$

$$= \underline{-101.867\ \text{KNm}}$$

$$\underline{M_{CB} = 0}$$

Set 2, 3, & 4 $\Rightarrow$ Similar problems with Data change diagrams of beam in below figure by slope deflection method?



Sol:

writing FEMs: $M_{FAB} = -\frac{20\times8^2}{12} = -106.67$ KNm

$M_{FBA} = \frac{20\times8^2}{12} = 106.67$ KNm

$M_{FBC} = -\frac{100\times1\times5^2}{6^2} = -69.444$ KNm

$M_{FCB} = \frac{100\times1^2\times5}{6^2} = 13.889$ KNm

Slope-deflection Equations: $M_{AB} = -106.67 + \frac{2EI}{L=8}(2\theta_A+\theta_B-0)$

$M_{BA} = 106.67 + \frac{2EI}{8}(\theta_A+2\theta_B-0)$

$M_{BC} = -69.444 + \frac{2EI}{6}(2\theta_B+\theta_C-0)$

$M_{CB} = 13.889 + \frac{2EI}{6}(\theta_B+2\theta_C-0)$

Substituting, $\theta_A = 0$; $\theta_C = 0$

Joint Equilibrium equation, $\Sigma M_B = 0$; $M_{BA} + M_{BC} = 0$

$106.67 + \frac{2EI}{8}(0+2\theta_B) - 69.444 + \frac{2EI}{6}(2\theta_B+0) = 0$

$37.226 + \frac{\cancel{4}EI\theta_B}{\cancel{8}2} + \frac{\cancel{4}^2EI\theta_B}{\cancel{6}3} = 0$ $\quad \theta_B = -\frac{31.908}{EI}$

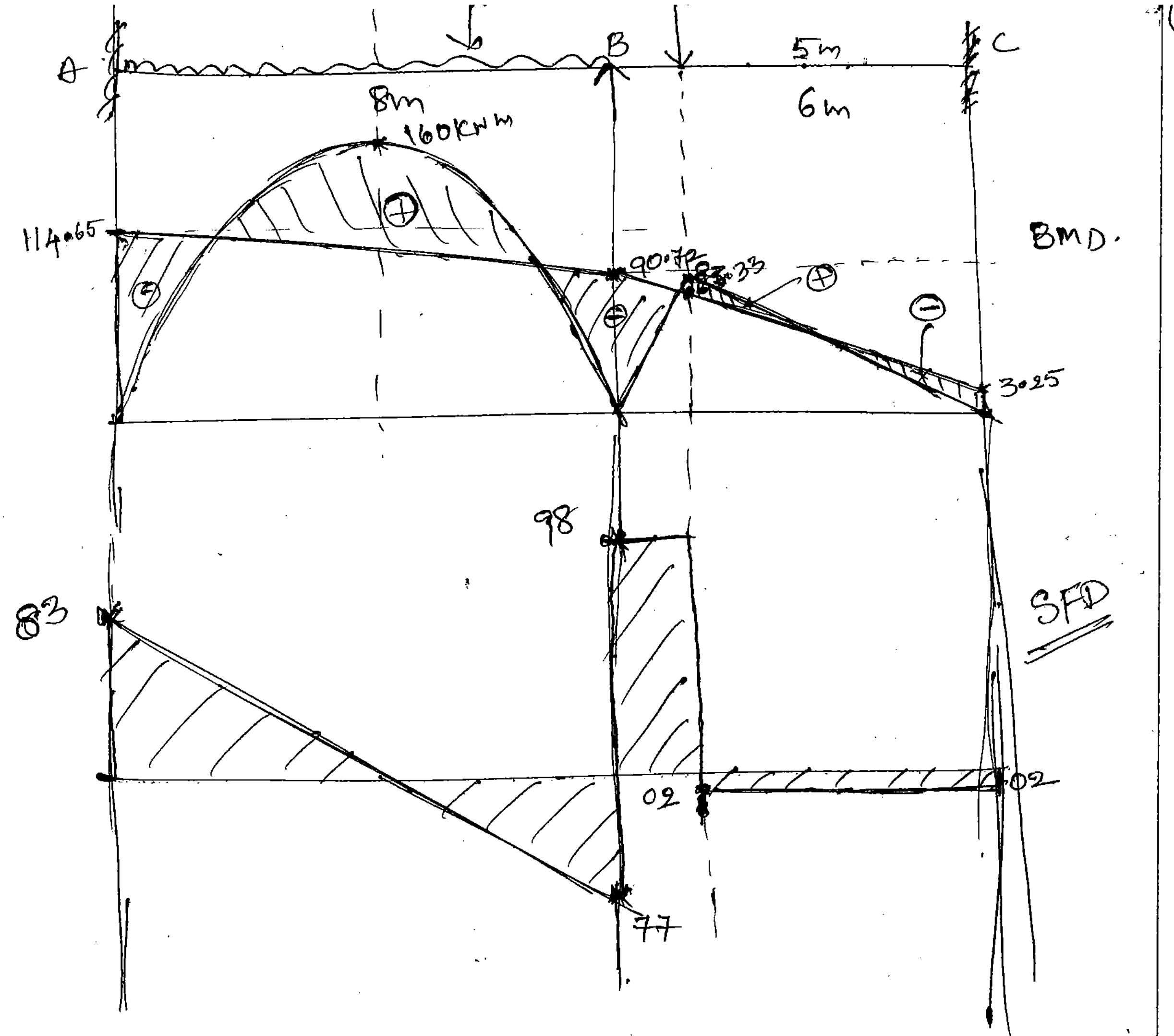
$EI\theta_B\left(\frac{1}{2}+\frac{2}{3}\right) = -37.226$

$\therefore \; M_{AB} = -106.67 + \frac{\cancel{2}\cancel{EI}}{\cancel{8}4}\left(0 - \frac{31.908}{\cancel{EI}}\right) = -114.647$ KNm

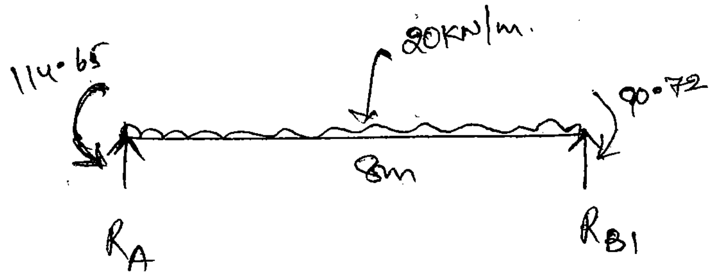
$M_{BA} = 106.67 + \frac{\cancel{2}EI}{\cancel{8}\cancel{4}2}\left(0 - \cancel{2}\times\frac{31.908}{EI}\right) = 90.716$ KNm

$M_{BC} = -69.444 + \frac{\cancel{2}\cancel{EI}}{\cancel{6}3}\left(\cancel{2}\times -\frac{31.908}{\cancel{EI}}\right) = -90.716$ KNm

$M_{CB} = 13.889 + \frac{\cancel{2}\cancel{EI}}{\cancel{6}3}\left(-\frac{31.908}{\cancel{EI}}\right) = 3.253$ KNm.



Shear force :- AB :-

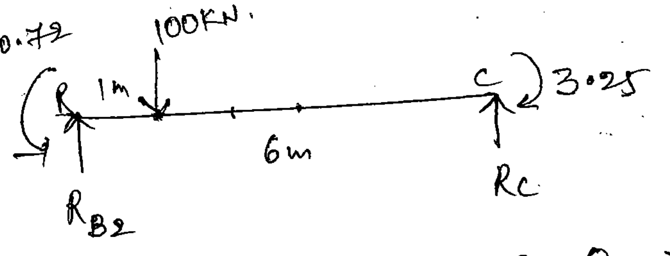


$\Sigma M_A = 0$

$114.65 + R_{B1} \times 8 - 90.72 - 20 \times 8 \times 4 = 0 \Rightarrow R_{B1} = 77\,\text{KN}$

$R_A = 83\,\text{KN}$

BC :-

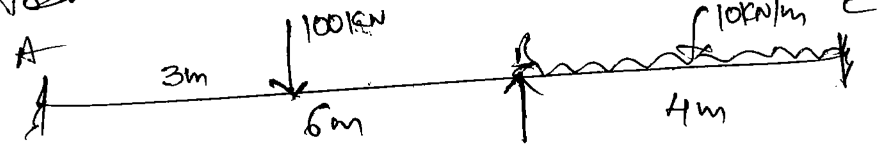


$\Sigma M_B = 0 \quad 90.72 + R_C \times 6 - 3.25 - 100 = 0 \Rightarrow R_C = 2.0\,\text{KN}$

$R_{B2} = 98\,\text{KN}$

Problem- ABC is ...
EI throughout its length. The end supports A and C are fixed and beam is continuous over middle support B. Span BC is uniformly loaded with 10 KN/m length while a concentrated vertical load of 100KN acts at the mid span of AB. Calculate the moments by slope deflection method?

Soln Assume span AB = 6m, BC = 4m



FEMs $M_{FAB} = -\frac{100 \times 6}{8} = -75$

$M_{FBA} = 75$

$M_{FBC} = -\frac{10 \times 4^2}{12} = -13.333$

$M_{FCB} = 13.333$

SDEs $M_{AB} = -75 + \frac{2EI}{6}(2\theta_A + \theta_B) = -75 + \frac{2EI}{6}(\theta_B)$

$M_{BA} = 75 + \frac{2EI}{6}(\theta_A + 2\theta_B) = 75 + \frac{2EI}{6}(2\theta_B)$

$M_{BC} = -13.333 + \frac{2EI}{4}(2\theta_B + \theta_C) = -13.333 + \frac{2EI}{4}(2\theta_B)$

$M_{CB} = 13.33 + \frac{2EI}{4}(\theta_B + 2\theta_C) = 13.33 + \frac{2EI}{4}(\theta_B)$

Joint Eqns $\Sigma M_B = 0$; $M_{BA} + M_{BC} = 0$

$75 + \frac{2EI}{6}(2\theta_B) - 13.333 + \frac{2EI}{4}(2\theta_B) = 0$

$EI\,\theta_B\left(\frac{2}{3} + 1\right) = -61.667$

$EI\,\theta_B = \frac{-61.667 \times 3}{5} = -37$

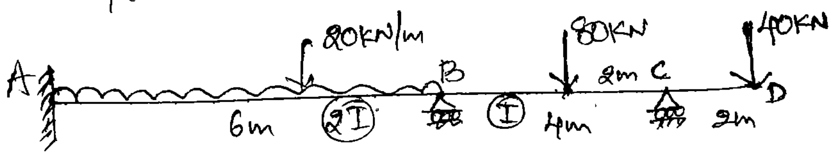
$\boxed{\theta_B = \frac{-37}{EI}}$

$\therefore M_{AB} = -75 + \frac{2EI}{6}\left(-\frac{37}{EI}\right) = -87.333$ KNM

$M_{BA} = 75 + \frac{2EI}{6}\left(2 \times -\frac{37}{EI}\right) = 50.333$ KNM

$M_{BC} = -13.333 + \frac{2EI}{4}\left(2 \times -\frac{37}{EI}\right) = -50.333$ KNM

$M_{CB} = 13.333 + \frac{2EI}{4}\left(-\frac{37}{EI}\right) = -5.167$ KNM by slope deflection method? and draw BMD?



Sol:

Fixed end moments:- $M_{FAB} = -\frac{20 \times 6^2}{8} = -60$ KNm

$M_{FBA} = 60$ KNm

$M_{FBC} = -\frac{80 \times 4}{8} = -40$ KNm

$M_{FCB} = 40$ KNm

Slope deflection Equations $M_{AB} = -60 + \frac{2E(2I)}{6}(2\theta_A + \theta_B - 0)$

$= -60 + \frac{2}{3} EI\theta_B \quad (\because \theta_A = 0)$

$M_{BA} = 60 + \frac{2E(2I)}{6}(\theta_A + 2\theta_B - 0)$

$= 60 + \frac{4}{3} EI\theta_B$

$M_{BC} = -40 + \frac{2EI}{4}(2\theta_B + \theta_C - 0) = -40 + EI\theta_B + 0.5EI\theta_C$

$M_{CB} = 40 + \frac{2EI}{4}(\theta_B + 2\theta_C - 0) = 40 + 0.5EI\theta_B + EI\theta_C$

Equilibrium Equations: (1) $\Sigma M_B = 0$; $M_{BA} + M_{BC} = 0$

$60 + \frac{4}{3}EI\theta_B + (-40) + EI\theta_B + 0.5EI\theta_C = 0$

$2.333 EI\theta_B + 0.5EI\theta_C = -20 \rightarrow$ (3)

(2) $\Sigma M_C = 0 \Rightarrow M_{CB} + M_{CD} = 0$ $\therefore M_{CB} = 80$ KNm

But, $M_{CD} = -40 \times 2 = -80$

$40 + 0.5EI\theta_B + EI\theta_C = 80$

$0.5EI\theta_B + EI\theta_C = 40$ — (4)

Solving (3) & (4) $\quad EI\theta_B = -19.2$

$EI\theta_C = 49.6$

Substituting θ_B, & θ_C in slope deflection equations we get final End moments

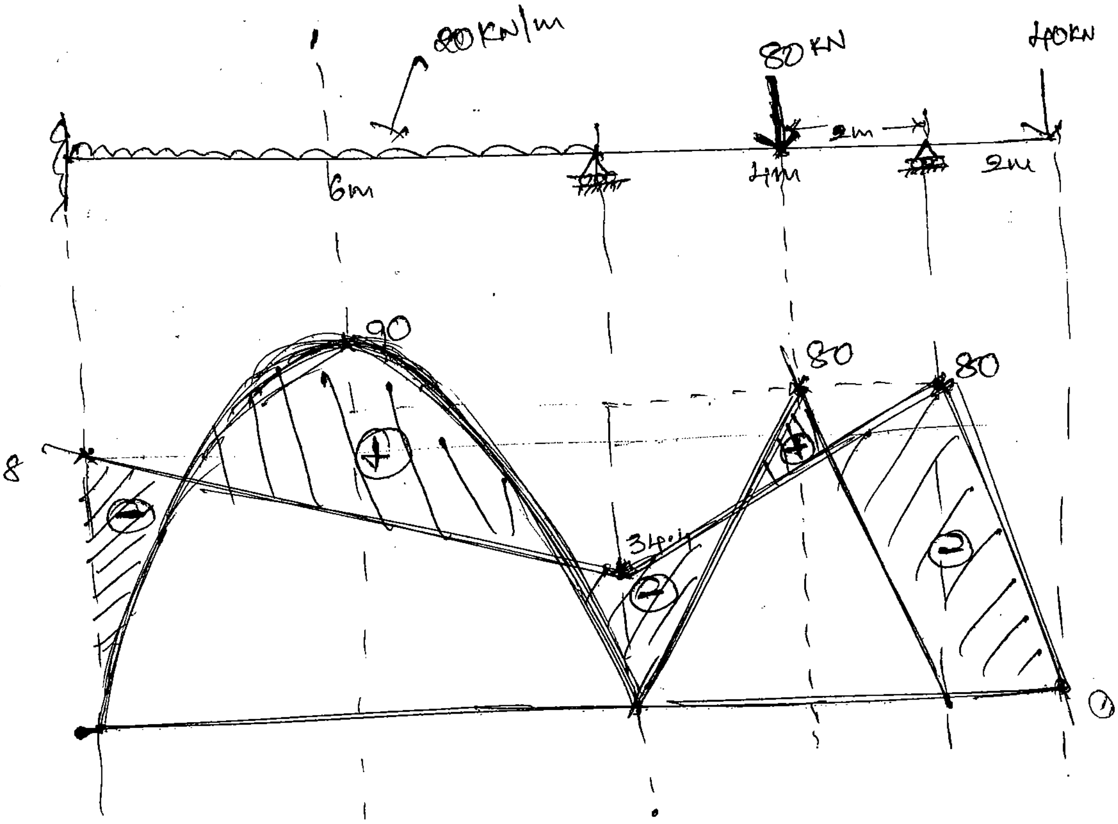
$$M_{AB} = -60 + \frac{8}{3}(-19.2) = -72.8 \text{ KNm}$$

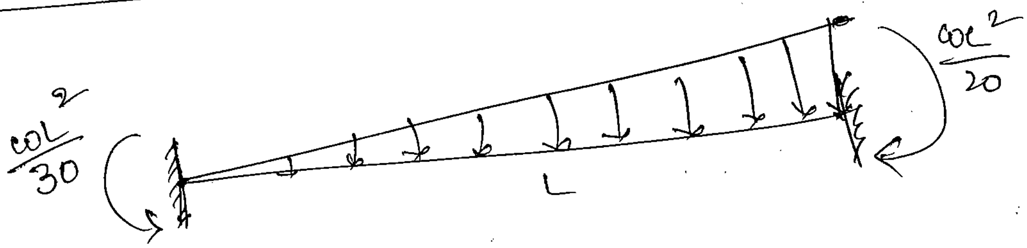
$$M_{BA} = 60 + \frac{4}{3}(-19.2) = 34.4 \text{ KNm}$$

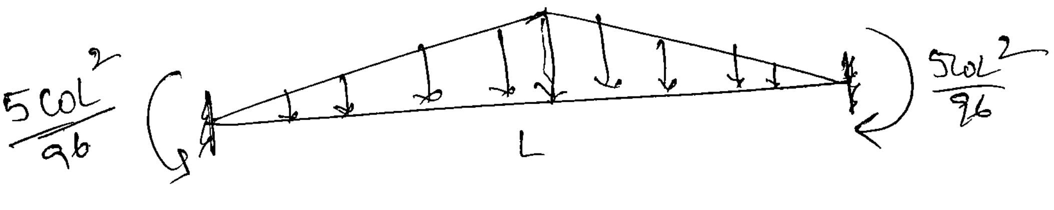
$$M_{BC} = -40 + (-19.2) + 0.5(49.6) = -34.4 \text{ KNm}$$

$$M_{CB} = 80 \text{ KNm}$$

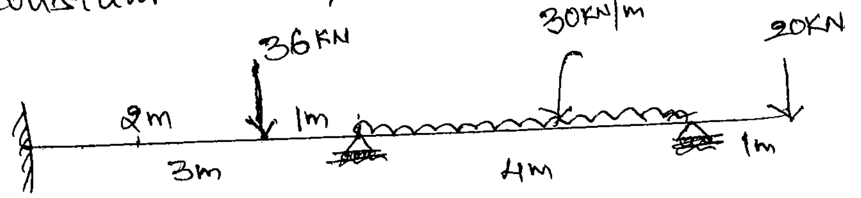
BMD







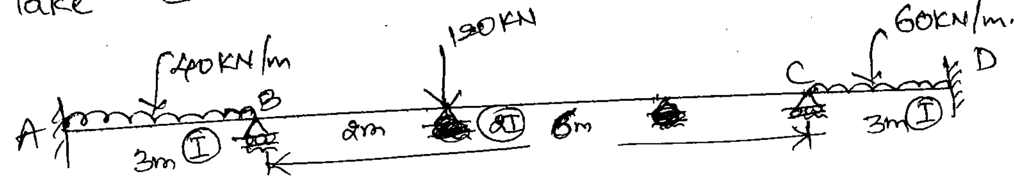
1. Analyse the continuous beam shown in fig below by slope deflection method and draw bending moment diagram. Flexural Rigidity is constant throughout?



Ans: $M_{AB} = 2.88$ KNm ; $M_{BA} = 37.76$ KNm ; $M_{BA} = -M_{BC}$

$M_{CB} = 20$ KNm.

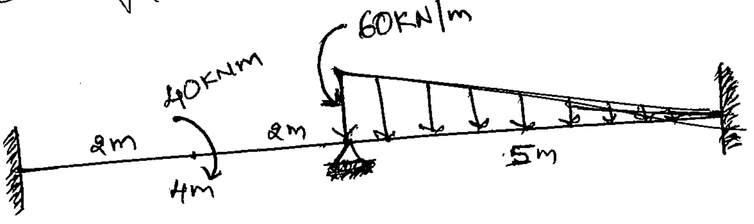
2. Analyse the continuous beam ABCD shown in figure given below, if support C sink by 10mm. Take $E = 2\times10^5$ N/mm^2 and $I = 4\times10^7$ mm^4.



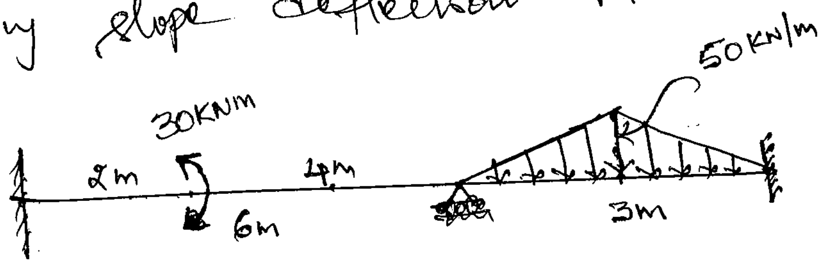
Ans:- $M_{AB} = -0.111$ KNm $M_{BA} = -M_{BC} = 89.777$ KNm ;

$M_{CB} = -M_{CD} = 24.111$ KNm ; $M_{DC} = 88.111$ KNm.

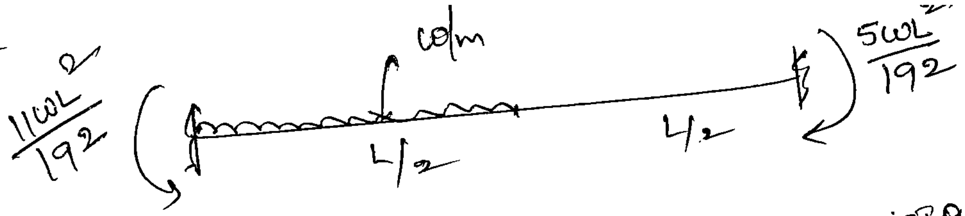
3. Analyse the continuous beam shown below using slope deflection method?

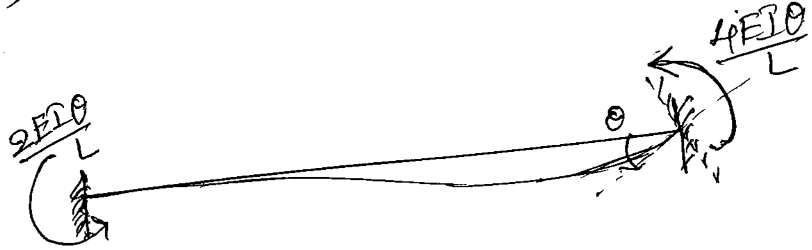


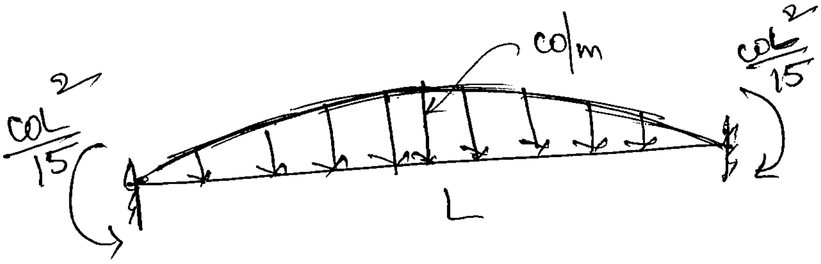
4. Analyse the continuous beam shown below using slope deflection method?

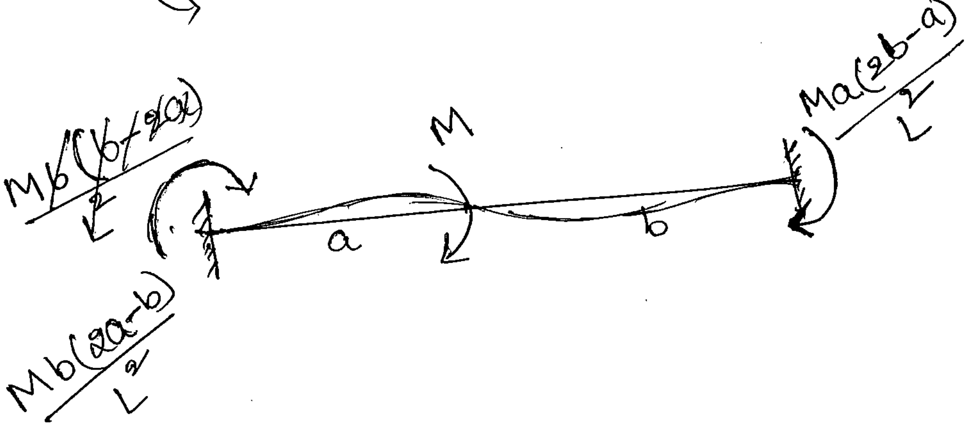


FEM's :-


11ωL²/192
ω/m
L/2
L/2
5ωL²/192


2EIθ/L
θ
4EIθ/L

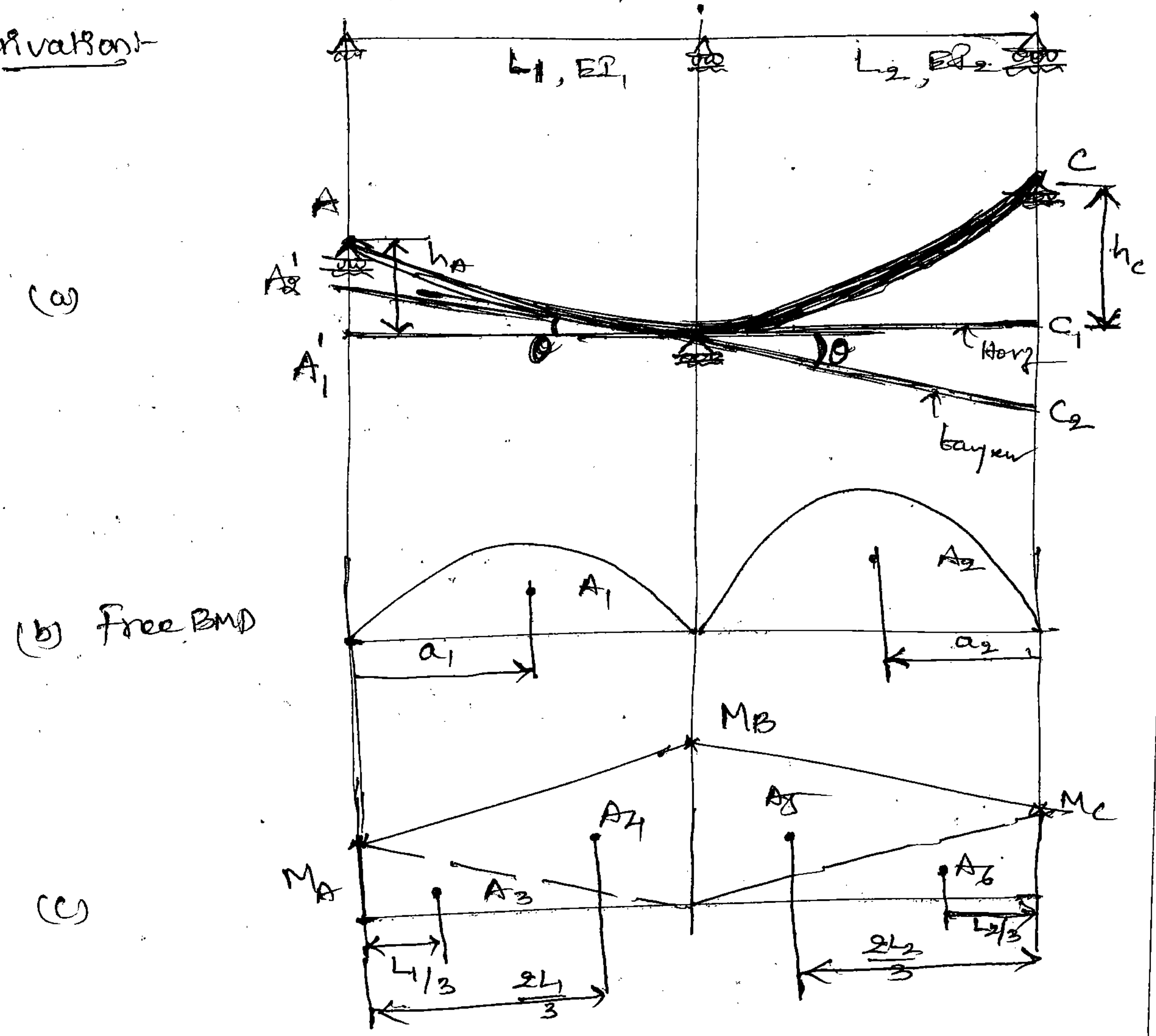

ω/m
ωL²/15
ωL²/15
L


Mb(b-2a)/L²
M
a
b
Ma(2b-a)/L²
Mb(2a-b)/L²

Continuous Beams

CLAPEYRON'S Theorem:- It is the relation between the moments at three successive supports. This is derived from the consistency condition that the slope at the middle support calculated from left span and the right span should be same.

Derivation:-



From (a) $\tan\theta = \dfrac{A_1'A_2'}{L_1} = \dfrac{C_1C_2}{L_2}$ —— ①

But $A_1'A_2' = AA_1' - AA_2' = h_A - AA_2'$

$= h_A -$ Vertical intercept of A from tangent at B

$= h_A -$ moment of (M/EI) diagram b/w B and A about A.

$$= h_A - \frac{1}{EI_1}\left[A_1 a_1 + A_3\frac{L_1}{3} + A_4 \cdot \frac{2L_1}{3}\right]$$

$$= h_A - \frac{1}{EI_1}\left[A_1 a_1 + \left(\frac{1}{2}\times M_A \times L_1\right)\frac{L_1}{3} + \frac{1}{2}\times M_B \times L_1 \times \frac{2L_1}{3}\right]$$

$$= h_A - \frac{1}{EI_1}\left[A_1 a_1 + \frac{M_A L_1^2}{6} + \frac{M_B L_1^2}{3}\right]$$

$$= h_A - \frac{1}{6EI_1}\cdot\left[6A_1 a_1 + M_A L_1^2 + 2M_B L_1^2\right] \quad \text{—} (2)$$

lly, $C_1 C_2 = CC_2 - CC_1$

= Vertical intercept of $\bar{C}$ from tangent at B $- h_c$

= moment of $\left(\frac{M}{EI}\right)$ b/w B and C about C $- h_c$

$$= \frac{1}{EI_2}\left[A_2 a_2 + A_5 \cdot \frac{2L_2}{3} + A_6 \cdot \frac{L_2}{3}\right] - h_c$$

$$= \frac{1}{EI_2}\left[A_2 a_2 + \frac{1}{2}\times M_B \times L_2 \times \frac{2L_2}{3} + \frac{1}{2}\times L_2 \times M_c \times \frac{L_2}{3}\right] - h_c$$

$$= \frac{1}{6EI_2}\left[6A_2 a_2 + 2M_B L_2^2 + M_c L_2^2\right] - h_c \quad \text{—} (3)$$

Substituting (2) & (3) in (1)

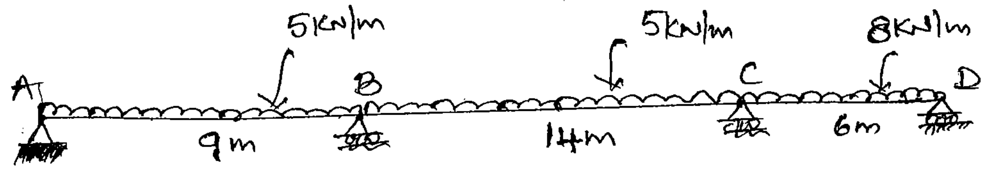
$$\frac{h_A}{L_1} - \frac{1}{6EI_1 L_1}\left[6A_1 a_1 + M_A L_1^2 + 2M_B L_1^2\right] = \frac{1}{6EI_2 L_2}\left[6A_2 a_2 + 2M_B L_2^2 + M_c L_2^2\right] - \frac{h_c}{L_2}$$

$\times 6E$

$$\frac{6Eh_A}{L_1} - \frac{1}{I_1 L_1}\left[6A_1 a_1 + M_A L_1^2 + 2M_B L_1^2\right] = \frac{1}{I_2 L_2}\left[6A_2 a_2 + 2M_B L_2^2 + M_c L_2^2\right] - \frac{6Eh_c}{L_2}$$

$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_c\left(\frac{L_2}{I_2}\right) = -\frac{6A_1 a_1}{I_1 L_1} - \frac{6A_2 a_2}{I_2 L_2} + \frac{6Eh_A}{L_1} + \frac{6Eh_c}{L_2} \text{ //}$$

Prob:- A continuous beam ABCD is simply supported over three spans. Span AB is 9m carrying an UDL of 5KN/m. Span BC is 14m carrying an UDL of 5KN/m and span CD is 6m carrying an UDL of 8KN/m. Find the moment over supports B and C. Draw BMD? (2)-III



Sol:- Applying three moment theorem for A, B & C supports

$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) = -\frac{6A_1a_1}{I_1L_1} - \frac{6A_2a_2}{I_2L_2} + \frac{6Eh_A}{L_1} + \frac{6Eh_C}{L_2}$$

$\because h_A = h_C = 0$

$\because$ EI is same throughout $I_1 = I_2 = I$

$$\therefore M_A L_1 + 2M_B(L_1+L_2) + M_C L_2 = -\frac{6A_1a_1}{L_1} - \frac{6A_2a_2}{L_2}$$

$$\cancel{M_A(9)}^{0} + 2M_B(9+14) + M_C(14) = -\left(6\times\frac{2}{3}\times 9\times\frac{5\times 9^2}{8}\times 4.5\right)/9 - \left(6\times\frac{2}{3}\times 14\times\frac{5\times 14^2}{8}\times 7\right)/14.$$

$$46M_B + 14M_C = -\frac{8201.25}{9} - \frac{48020}{14}$$

$$46M_B + 14M_C = -911.25 - 3430 = -4341.25 \quad \text{--- (1)}$$

Applying three moment theorem for B, C, D supports

$$M_B L_1 + 2M_C(L_1+L_2) + M_D(L_2) = -\frac{6A_1a_1}{L_1} - \frac{6A_2a_2}{L_2}$$

$$14M_B + 2M_C(14+6) + M_D(6) = -3430 - \frac{6\times\frac{2}{3}\times 6\times\frac{8\times 6^2}{8}\times 3}{6}$$

$$= -3430 - 432 = -3862$$

$$14M_B + 40M_C + \cancel{M_D(6)}^{0}$$

$$14M_B + 40M_C = -3862 \quad \text{--- (2)}$$

Solving (1) & (2)

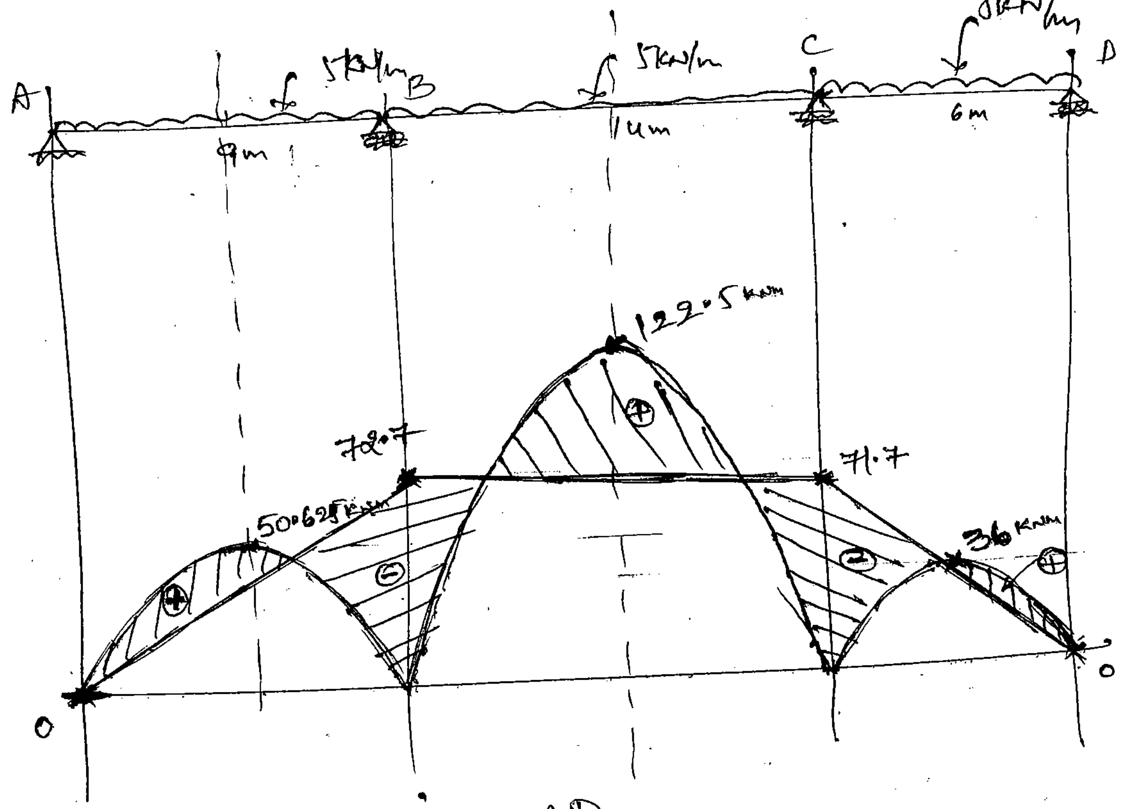
$-M_B + 0.304 M_C = +94.575$

$M_B + 2.857 M_C = -275.852$

$2.5526\, M_C = -181.482$

$\therefore M_C = -71.01$ KNm.

$M_B = -72.7$ KNm.

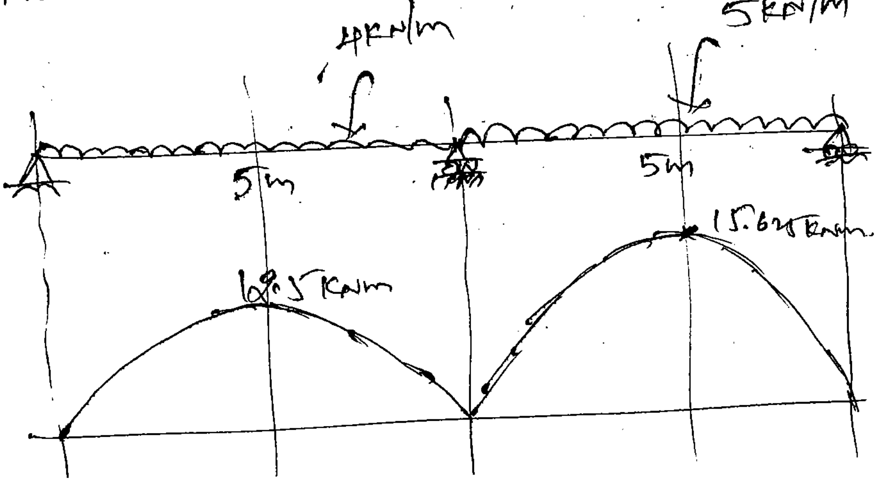


BMD

2016 Set-2

prob:- A continuous beam ABC of length 10m is simply supported, AB and BC is of 5m length each. Span AB carries an UDL of 4 KN/m and BC carries a UDL of 5 KN/m. Support B sinks down by 5mm below the supports A & C. M.I of beam is $10^8\ mm^4$ and E is 180 KN/mm. Find the support moments and draw BMD?

Sol:



Applying Clapeyron's Theorem

$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) = -\frac{6A_1a_1}{L_1I_1} - \frac{6A_2a_2}{L_2I_2} + \frac{6Eh_A}{L_1} + \frac{6Eh_C}{L_2}$$

∵ EI is same throughout,

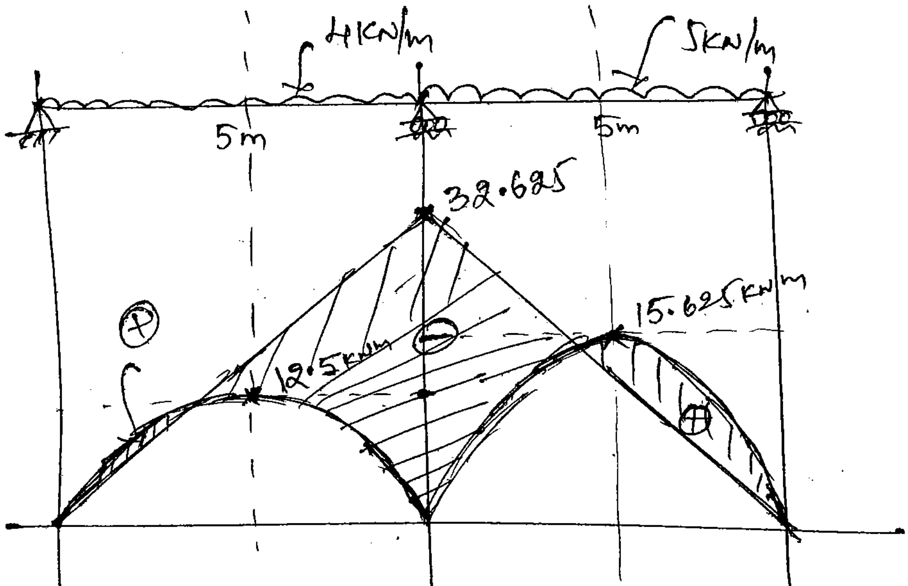
$$\cancel{M_A}^{0}(L_1) + 2M_B(L_1+L_2) + \cancel{M_C(L_2)}^{0} = -\frac{6A_1a_1}{L_1} - \frac{6A_2a_2}{L_2} + \frac{6EIh_A}{L_1} + \frac{6EIh_C}{L_2}$$

$$2M_B\times 10 = -\frac{6\times\frac{2}{3}\times5\times12.5\times2.5}{5} - \frac{6\times\frac{2}{3}\times5\times15.625\times2.5}{5} + \frac{6\times180\times10^{8}\times10^{-6}\times\left(\frac{5}{1000}\right)}{5} + \frac{6\times180\times10^{8}\times10^{-6}\times\left(\frac{8}{1000}\right)}{5}$$

$= -125 - 156.25 + 108 + 108$

$= -65.25 \Rightarrow M_B = \underline{\underline{-32.625\ \text{KNm}}}$ $/10 = -3.2625$ KNm

BMD:-

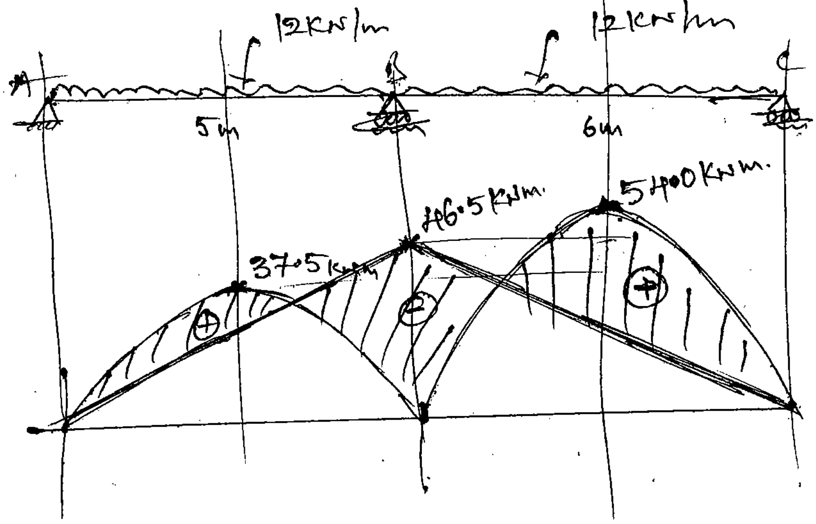


2016 Set-3:- Three moment theorem Derivation?

Set-4:- probl: A continuous beam ABCD is SS over three spans. AB is 8m carrying UDL of 4KN/m, BC is 12m carrying UDL of 3KN/m and CD is 5m carrying UDL of 6KN/m. Find the moments over supports B and C. Draw BMD?

at A and C and continuous over support B with AB = 5m and BC = 6m. A UDL of 12 KN/m is acting over the beam. The moment of inertia is I throughout the span. Analyse the continuous beam and draw SFD and BMD?

Sol:



Applying clapeyron's eqn:-

$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) = -\frac{6A_1a_1}{I_1L_1} - \frac{6A_2a_2}{I_2L_2} + \frac{6Eh_A}{L_1} + \frac{6Eh_C}{L_2}$$

∵ EI is same throughout; $h_A = h_C = 0$

$M_A = M_C = 0$ (Simple ends)

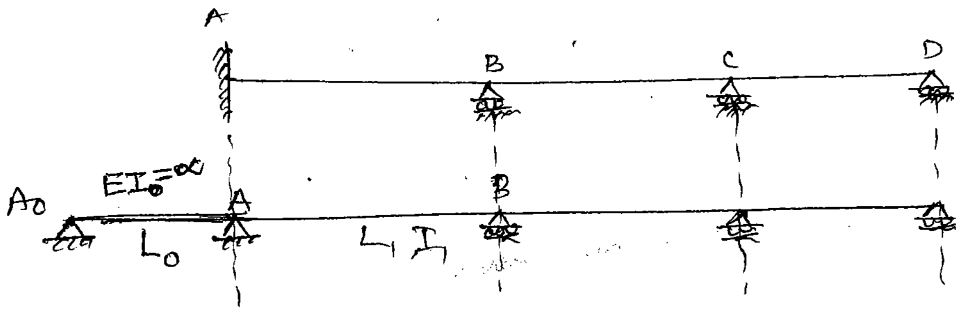
$$\therefore 2M_B(L_1+L_2) = -\frac{6A_1a_1}{L_1} - \frac{6A_2a_2}{L_2}$$

$$2M_B(5+6) = -\frac{6\times\frac{2}{3}\times5\times37.5\times2.5}{5} - \frac{6\times\frac{2}{3}\times6\times54\times3}{6}$$

$$22M_B = -375 - 648 = -1023$$

$$\therefore M_B = 46.5\ \text{KNm}$$

Three moment equation to problems with fixed end :-



* Add an imaginary span AA_0 of length L_0 with infinite flexural Rigidity, $EI_0 = \alpha$

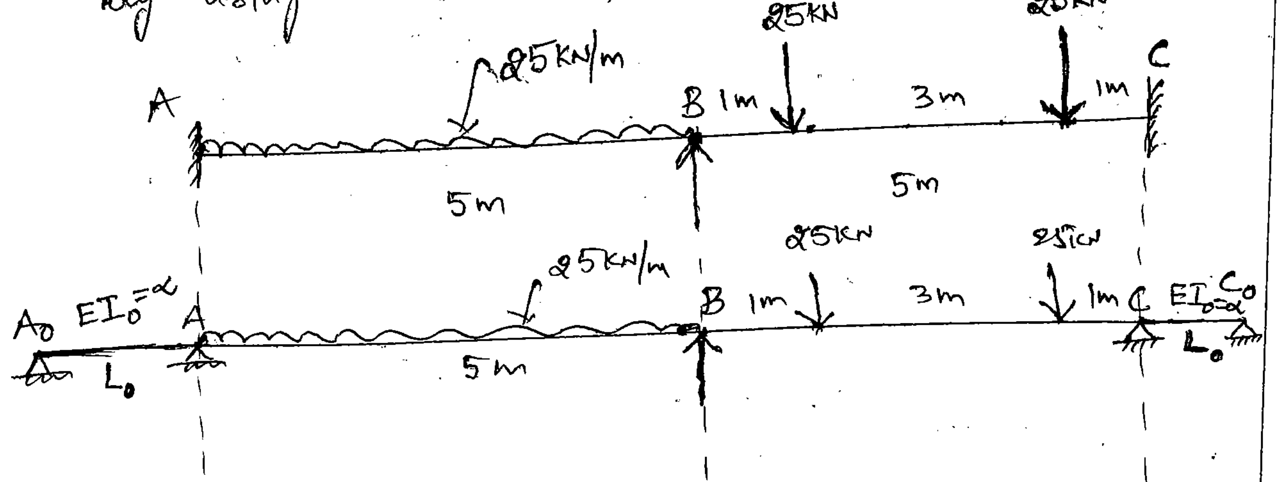
$$\therefore \; M_0\cancel{\left(\frac{L_0}{\alpha}\right)} + 2M_A\left(\cancel{\frac{L_0}{\alpha}} + \frac{L_1}{I_1}\right) + M_B\left(\frac{L_1}{I_1}\right)$$

$$= -\cancel{\frac{6A_0 a_0}{\alpha L_0}} - \frac{6A_1 a_1}{I_1 L_1} + \cancel{\frac{6Eh_0}{L_0}} + \frac{6Eh_b}{L_1}$$

$$\therefore \; 2M_A\left(\frac{L_1}{I_1}\right) + M_B\left(\frac{L_1}{I_1}\right) = -\frac{6A_1 a_1}{I_1 L_1} + \frac{6Eh_b}{L_1}$$

Supply Dec/Jan 15-16

probl- Solve the continuous beam in below figure by using theorem of three moments?



Applying the three moment equation for A_0, A & B supports

$$\therefore \; M_0\cancel{\left(\frac{L_0}{I_0}\right)} + 2M_A\left(\cancel{\frac{L_0}{I_0}} + \frac{L_1}{I_1}\right) + M_B\left(\frac{L_1}{I_1}\right) = -\cancel{\frac{6A_0 a_0}{I_0 L_0}} - \frac{6A_1 a_1}{I_1 L_1}$$

$$+ \cancel{\frac{6Eh_0}{L_0}} + \frac{6Eh_b}{4}$$

$(\because h_b = 0)$

$$2M_A(L_1) + M_B(L_1) = -\frac{6A_1 a_1}{L_1} \Rightarrow (2M_A + M_B)\cancel{5} = -\frac{\cancel{6}^{2} \times \frac{2}{\cancel{3}} \times \cancel{5} \times \left(\frac{25 \times 5^{\cancel{2}}}{8}\right) \times 2.5}{\cancel{5}}$$

$$2M_A + M_B = -156.25 \longrightarrow ①$$

Applying three moment equation to A, B & C supports.

$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) = -\frac{6A_1\bar{a}_1}{I_1L_1} - \frac{6A_2\bar{a}_2}{I_2L_2} + \frac{6E\cancel{h_a}^{0}}{L_1} + \frac{6E\cancel{h_c}^{0}}{L_2}$$

$\because h_a = 0 = h_c$; $I_1 = I_2 = I$.

$$M_A(5) + 2M_B(5+5) + M_C(5) = -\frac{6A_1\bar{a}_1}{L_1} - \frac{6A_2\bar{a}_2}{L_2}$$

$$M_A 5 + 20M_B + 5M_C = -6\times\frac{2}{3}\times 5\times\frac{(25\times5^2)}{8}\times\frac{2.5}{5}$$

$$-\frac{6\left[(1\times25)+(25\times3)\right]\times2.5}{5}$$

$$5M_A + 20M_B + 5M_C = -781.25 - 300 = -1081.25$$

$$M_A + 4M_B + M_C = -216.25 \quad \text{— (2)}$$

Applying three moment equation to B, C & C_0 Supports

$$M_B\left(\frac{L_2}{I_2}\right) + 2M_C\left(\frac{L_2}{I_2}+\cancel{\frac{L_0}{I_0}}^{0}\right) + M_{C_0}\cancel{\left(\frac{L_0}{I_0}\right)}^{0} = -\frac{6A_2\bar{a}_2}{I_2L_2} - \cancel{\frac{6A_0\bar{a}_0}{L_0I_0}}^{0} + \frac{6Eh_b}{L_2} + \frac{6E\cancel{h_0}^{0}}{L_0}$$

$(\because h_b = 0 ; I_2 = I)$

$$M_B(5) + 2M_C(5) = -\frac{6A_2\bar{a}_2}{L_2} = -300$$

$$M_B + 2M_C = -60 \quad \text{— (3)}$$

$$2M_A + M_B + 0 = -156.25 \quad \text{— (1)}$$

$$\underline{\cancel{2M_A} + 2M_B + 2M_C = +216.25}$$

$$(-)\cancel{2M_A} (-) 2M_B (-) 2M_C = -432.5$$

$$\cancel{2M_A} + 8M_B + \cancel{2M_C} = -216.25 \Rightarrow$$

$$6M_B = -216.25$$

$M_A = -60.104$ KNm.

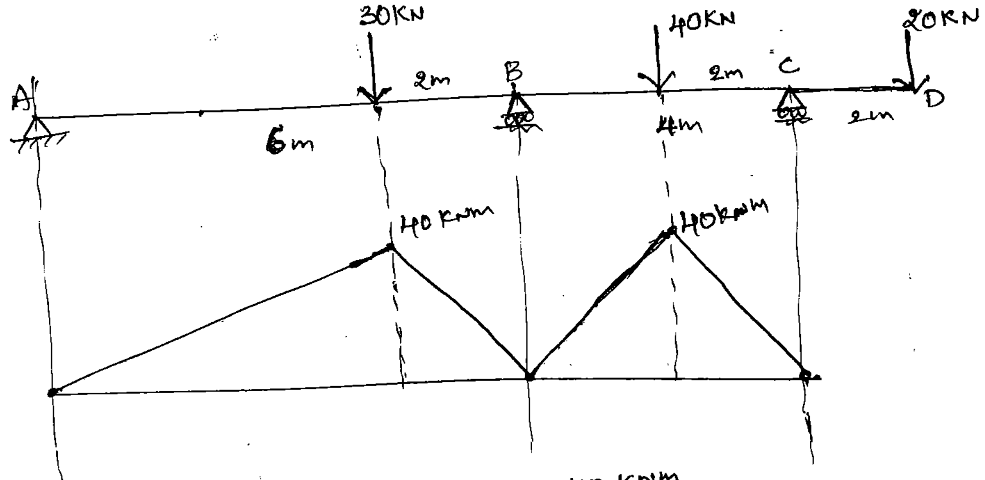
$M_B = -36.042$ KNm

$M_C = -11.98$ KNM

$\therefore M_A = -60$ KNm ; $M_B = -36$ KNm ; $M_C = -12$ KNM

5-III

Prob:- Analyse the continuous beam ABCD shown below, if support C settles down by 5mm. Take $E = 15\ KN/mm^2$. Moment of Inertia is constant throughout and is equal to $5\times10^9\ mm^4$



$M_A = 0$; $M_c = 20\times2 = -40\ KNm$

Applying three moment Equation,

$$0 + 2M_B(6+4) + M_c L_2 = \frac{-6A_1\bar{a}_1}{L_1} - \frac{6A_2\bar{a}_2}{L_2} + \cancel{\frac{6EIh_a}{L_1}}^{\,0} + \frac{6EIh_c}{L_2}$$

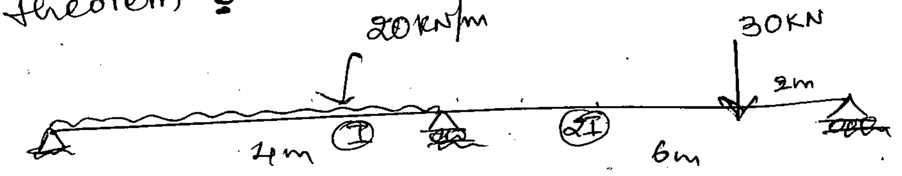
$(h_c = -5)$

$$20M_B - 40\times4 = -6\times\frac{\left(\frac{1}{2}\times6\times40\right)\times\frac{(6+4)}{3}}{6} - \frac{6\times\left(\frac{1}{2}\times4\times40\right)\times2}{4} - \frac{6\times15\times10^6\times5\times10^{-3}\times5\times10^{-3}}{4}$$

$M_B = -52.125\ KNm$

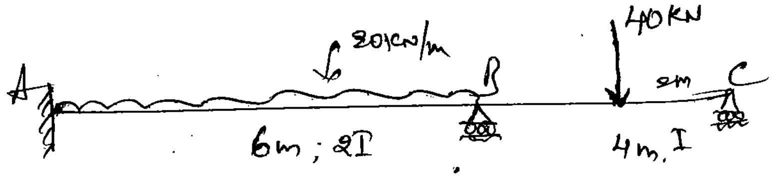
$\therefore\ M_A = 0$; $M_B = -52.125\ KNm$, $M_c = -40\ KNm$

SSBKT:- prob 8:- Analyse the two span continuous beam shown below using three moment theorem?



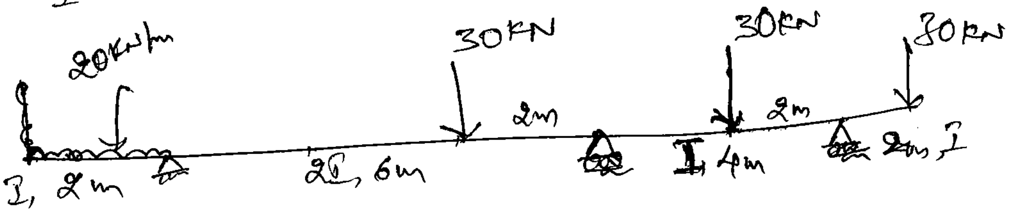
Ans! $M_B = -34.286\ KNm$.

SSBT1 Analyse the Continuous beam shown by three moment equation? Draw BMD?

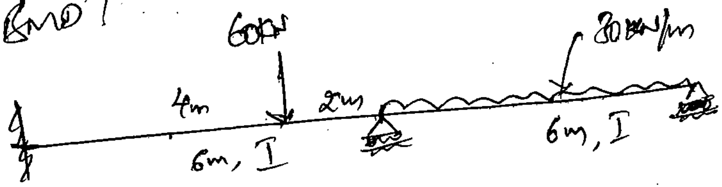


Ans: $M_A = -69.6$ kNm ; $M_B = -40.8$ kNm

SSBT Analyse the Continuous beam ABCDE shown below if support C sinks by 8mm. Given, $E = 200\ kN/mm^2$; $I = 0.8 \times 10^8\ mm^4$?. Ans: ($M_C = -1.429$ kNm)

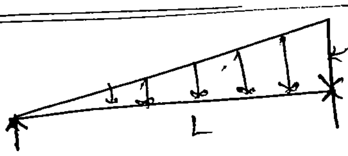


SSBT Analyse the beam ABC shown below and draw BMD?

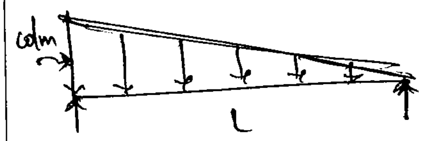


$M_A = -103.33$ kNm

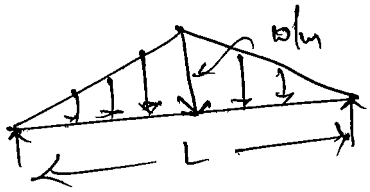
$M_B = -100$ kNm



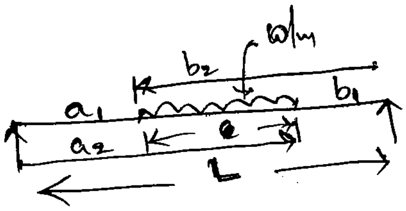
$\frac{6A_1a_1}{L} = (8/60)\,wL^3$ $\frac{6A_2a_2}{L} = (7/60)\,wL^3$



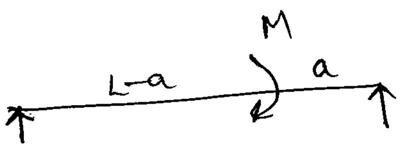
$\frac{6A_1a_1}{L} = (7/60)\,wL^3$ $\frac{6A_2a_2}{L} = (8/60)\,wL^3$



$\left(\frac{5}{32}\right) wL^3$ $\left(\frac{5}{32}\right) wL^3$



$\frac{w}{4L}\left[a_2^2(2L^2-a_2^2) - a_1^2(2L^2-a_1^2)\right]$ $\frac{w}{4L}\left[b_2^2(2L^2-b_2^2) - b_1^2(2L^2-b_1^2)\right]$



$-\left(\frac{M}{L}\right)\left[3(L-a)^2-L^2\right]$ $\left(\frac{M}{L}\right)\left[(3a^2-L^2)\right]$

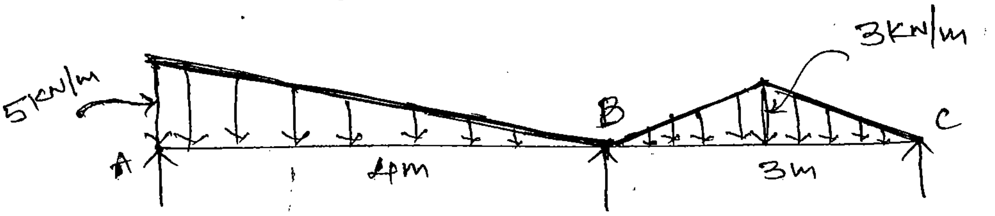
Continuous Beams

Exercise Questions from T.S.TM

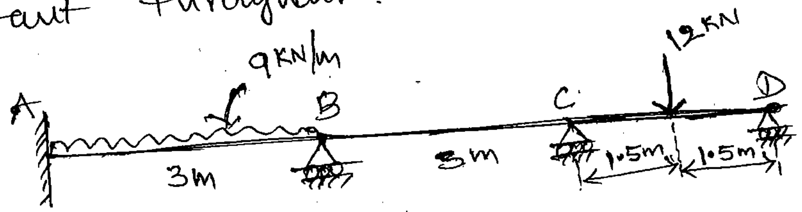
1. Define a continuous beam?

2. Derive Clapeyron's theorem of three moments including the effect of support settlements

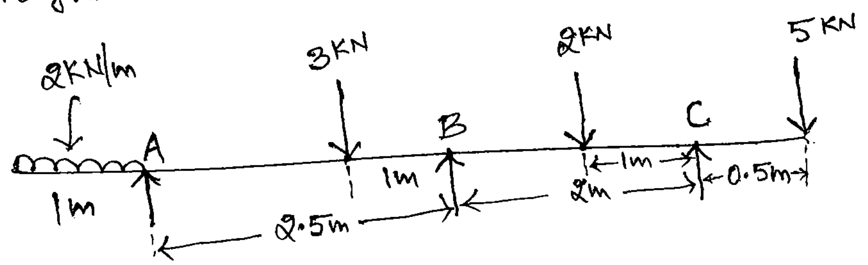
3. Analyse the beam shown below with constant EI using Clapeyron's three-moment equation?



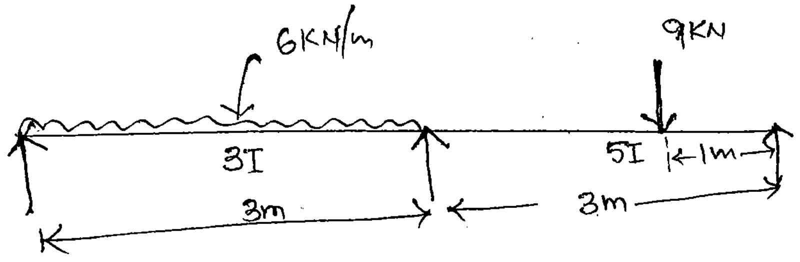
4. Using Clapeyron's theorem, solve the problem of continuous beam shown below. EI is constant throughout?



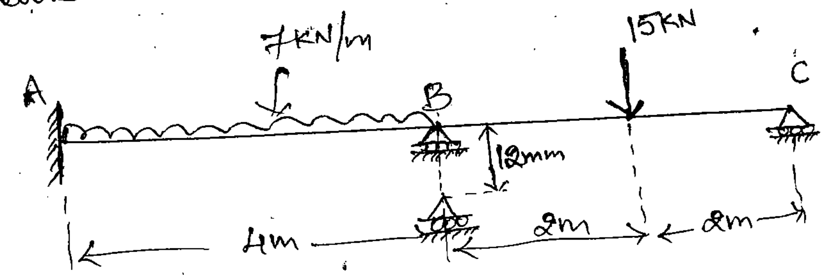
5. A continuous beam has overhangs on both sides as shown below. Apply three moment equation to determine the support moments. EI is constant throughout



(6) The moment of Inertia of a continuous beam is different for different spans as shown below. find the reactions ?



(7) The support 'B' of a continuous beam shown below has settled by 12mm. find out the moments at supports ?



(8) Draw the SFD and BMD for the continuous beam shown below ?